(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9610	Roll No.										

B. Tech.

(SEM. II) THEORY EXAMINATION 2010-11

MATHEMATICS—II

Time : 3 Hours *Total Marks : 100*

SECTION—A

Note : Attempt **all** questions. Each question carries equal marks.

(2×10=20)

1. (a) The roots of the auxiliary equation of the differential equation $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = 4e^{3t}$ are :

(i) 3, –3 (ii) 3, 3

(iii) –3, –3 (iv) None of those

(b) Particular integral of $(D^2 - 4D + 4)\, y = \sin 2x$ is ---- .

(c) Indicate True or False for the following statements :

(i) $J_n(x) = 0$ has repeated root at $x = 0$.

(ii) $J_{1/2}(x) = \sqrt{\dfrac{2}{\pi x}}\cos x$

(d) If $3x^2 = m\,P_2(x) + n\,P_0(x)$, then (m, n) are :

(i) (1, 2) (ii) (1, 3)

(iii) (2, 1) (iv) (2, 2)

(e) Indicate True and False for the following :

(i) Laplace Transform of a function exists if it is of exponential order.

(ii) $L\{u(t-a)\,f(t)\} = e^{-as}\,L\{f(t+a)\}$.

(f) $L^{-1}\left(\dfrac{2}{(s-1)(s-2)}\right)$ is ..... .

(g) Fill in the blanks :

(i) The product of two odd function is an ________ function.

(ii) sin nx is a periodic function with period ________ .

(h) State True and False in the following statements :

(i) Fourier series expansion of an odd function in (0, 2L) has only sine terms.

(ii) If $f(x) = x^2$ is expanded in a Fourier series in $(-\pi, \pi)$ then $b_n = 0$.

(i) The equation of steady state heat conduction in the rectangular-plate is ________ .

(j) The differential equation $Z_{xx} + x^2 Z_{yy} = 0$ is classified as :

(i) Hyperbolic (ii) Parabolic

(iii) Elliptic (iv) None of these.

SECTION—B

Note : Attempt any **three** parts from this sectoin. Each part carries equal marks. **(10×3=30)**

2. (a) Solve $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 10y + 37\sin 3x = 0$

and find the value of y when $x = \dfrac{\pi}{2}$ being given that y = 3, $\dfrac{dy}{dx} = 0$ when x = 0.

(b) Solve in series the differential equation

$$2x^2y'' + xy' - (x+1)y = 0$$

(c) Solve the initial value problem using Laplace transform

$\dfrac{d^2y}{dt^2} + 9y = r(t)$ with initial conditions y(0) = 0 and

$y'(0) = 4$ where $r(t) = \begin{cases} 8\sin t, & 0 < t < \pi \\ 0, & t < \pi \end{cases}$

(d) Find the Fourier series for the function

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 1-x, & 1 < x < 2 \end{cases}$$

Hence deduce that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$.

(e) Find the deflection u(x, y, t) of the tightly stretched rectangular membrane with sides a and b having wave velocity c = 1 if the initial velocity is zero and the initial deflection is $f(x, y) = \sin\frac{2\pi x}{a}\sin\frac{3\pi y}{b}$.

SECTION—C

Note :– Attempt any **two** parts from each question of this section. Each part carries equal marks. **(5×2×5=50)**

3. (a) Solve $(3x+2)^2\frac{d^2y}{dx^2} - (3x+2)\frac{dy}{dx} - 12y = 6x$.

(b) Solve the following simultaneous differential equations :

$$\frac{dx}{dt} = -4(x+y), \ \frac{dx}{dt} + 4\frac{dy}{dt} = -4y$$

with conditions $x(0) = 1, \ y(0) = 0$.

(c) Solve $\frac{d^2y}{dx^2} - \cot x\frac{dy}{dx} - y\sin^2 x = \cos x - \cos^3 x$.

4. (a) Obtain the first five terms in the expansion of following function in terms of Legendre polynomials :

$$f(x) = \begin{cases} 0, & -1 < x < 0 \\ 1, & 0 < x < 1 \end{cases}$$

(b) Show that

$$J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(n\phi - x\sin\phi)\,d\phi \text{ , n being positive integer.}$$

(c) (i) Prove that $\frac{d}{dx}\left[x^n J_n(x)\,dx\right] = x^n J_{n-1}(x)$

(ii) Evaluate $\int x^4 J_1(x)\,dx$.

5. (a) Use Laplace transform to evaluate :

$$\int_0^{\infty} \frac{e^{-at} - e^{-bt}}{t}\,dt.$$

(b) Find the inverse Laplace transform of

(i) $\frac{e^{-2\pi s}}{s(s^2+1)}$

(ii) $\dfrac{s}{s^2+6s+25}$.

(c) Draw the graph and find the Laplace transform of the following function of period 2a :

$$f(t)=\begin{cases}\dfrac{h}{a}t & , \quad 0<t<a\\ \dfrac{h}{a}(2a-t) & , \quad a<t<2a.\end{cases}$$

6. (a) Find the half range Fourier sine series of f(x) defined over the range $0<x<4$ as

$$f(x)=\begin{cases}x & , \quad 0<x<2\\ 4-x, & 2<x<4.\end{cases}$$

(b) Solve $yp+xq=xyz^2(x^2-y^2)$.

(c) Solve the partial differential equation

$$(D^2-DD'-2D'^2+2D+2D')Z=\sin(2x+y)$$

7. (a) Use method of separation of variables to solve

$$y^3\frac{\partial u}{\partial x}+x^2\frac{\partial u}{\partial y}=0$$

(b) Find the temperature distribution in a rod of length L whose end points are maintained at temperature zero and the initial temperature distribution is f(x).

(c) Find the possible general solutions of two dimensional Laplace equation using method of separation of variables.